

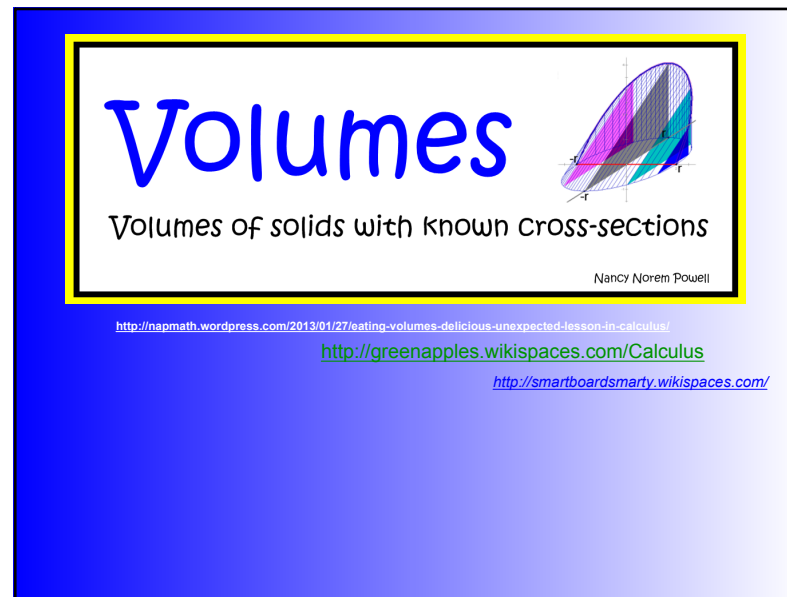
Eating

VOLUMES

in Calculus!

Nancy Norem Powell

Nov 15-7:27 AM



Volumes

Volumes of solids with known cross-sections

Nancy Norem Powell

<http://napmath.wordpress.com/2013/01/27/eating-volumes-delicious-unexpected-lesson-in-calculus/>

<http://greenapples.wikispaces.com/Calculus>

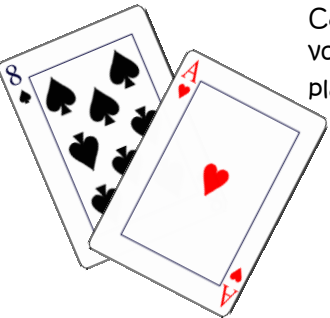
<http://smartboardsmarty.wikispaces.com/>

Nov 15-6:05 PM

Visualize a card from a deck of playing cards.


Can you find the area of a single playing card?

How about it's volume?



Can you find the volume of a deck of playing cards?

Oct 13-12:36 AM



Do you know something that has the same volume as this stack of CD's?

This is a stack of CD's. What information do you need to know to be able to set up an integral and calculate the volume of a solid formed this way.

Nov 15-9:39 PM



1. Given half of an orange, if you were going to identify a "base", what would it be?
2. Slice your 1/2 of the orange so that you get cross sections perpendicular to that base. What shape are the cross - sections?

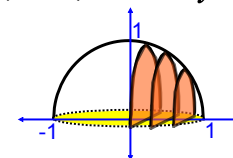
Orange 1

What do you know about a circle?

$$\text{Area of a circle} = \pi r^2$$

$$\text{Area of a semicircle} = \frac{\pi r^2}{2}$$

$$\text{Equation of a circle: } x^2 + y^2 = r^2$$



Orange 2

Area of a circle =

Area of a semicircle =

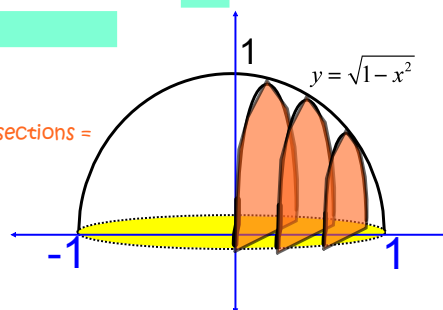
Equation of a circle:

Radius of the circle?

Radius of the semicircular cross-sections =

How many semicircles are there from $x = -1$ to $x = 1$?

How thick is each one?



Move the rectangles to reveal the answers.

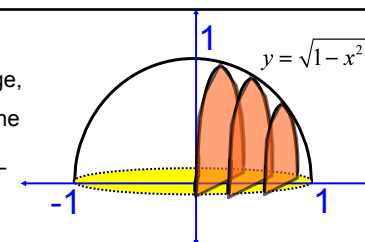
Orange 3

Move the rectangles to reveal the answers.

To find the volume of 1/2 of the orange, sum the of all of the from to .

$$V = \int_a^b \text{Area}(x) dx$$

$$V = \int_{-1}^1 \frac{\pi(r)^2}{2} dx$$



Area of a circle = πr^2

Area of a semicircle = $\frac{\pi r^2}{2}$

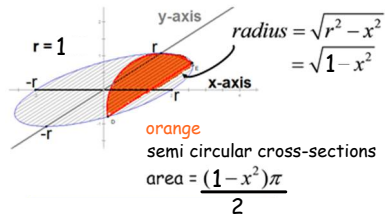
Equation of a circle: $x^2 + y^2 = r^2$

Radius of the circle = 1

Radius of the semicircles = $\sqrt{1 - x^2}$

Orange 4

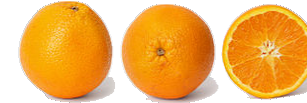
The area function $A(x)$ was written in terms of x and is called the integrand. The limits of integration showed where the square started and where it ended up as it moved along the x -axis. The dx represented the infinitely thin thickness of each cross-section. The integral sums those areas calculating the volume of the solid. I realize that it might be a bit simplified but it made the calculus make sense - YAY! There are sites on the Internet that will do the integration like the [Definite Integral Calculator](#). Take the mystery out of the notation!



$$V = \int_{-1}^1 \frac{\pi(\sqrt{1-x^2})^2}{2} dx$$

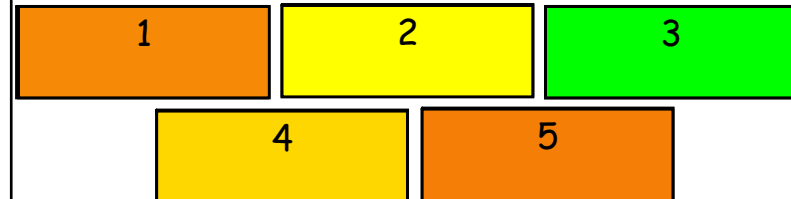
Oct 13-12:42 AM

Move the rectangles to reveal the answers.



Radius of the semicircles = $\sqrt{1-x^2}$
 Area of a semicircle = $\frac{\pi r^2}{2}$

Can you find more than one integral that will find the volume of $\frac{1}{2}$ the orange using known cross-sections.

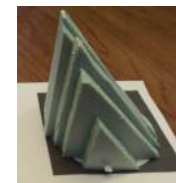


Nov 15-9:38 PM



$A_{\text{square}} = \text{side}^2$
 $A_{\text{triangle}} = \frac{1}{2}(\text{base})(\text{height})$
 $A_{\text{isos rt triangle}} = \frac{1}{2}(\text{leg})^2$
 $A_{\text{equilateral triangle}} = \frac{\text{side}^2 \sqrt{3}}{4}$
 $A_{\text{trapezoid}} = \frac{1}{2}h(\text{base}_1 + \text{base}_2)$
 $A_{\text{circle}} = \pi r^2$

Oct 13-1:09 AM



Isosceles Right Triangular Cross-sections on a circular base



Equilateral Triangular cross-sections on a circular base

Oct 13-1:10 AM

Volumes on Base.gsp

Start with the *circle base with squares* investigation.

a) Sketch a picture of the solid formed.

b) Write the integral that helps you find the volume of the solid when $r = 2$.

$V =$

Start with the *Circle base with equilateral triangles* investigation.

a) Sketch a picture of the solid formed.

b) Write the integral that helps you find the volume of the solid when $r = 2$.

$V =$

Nov 15-8:02 AM

Isosceles
Right
Triangles

Click to reveal
the integral

Semicircles

Click to reveal
the integral

Right Triangles

Click to reveal the
integral

Pictures made with Geometer's
Sketchpad and Calculus in Motion
by Audrey Weeks <http://www.calculusinmotion.com/>

Nov 15-5:58 PM

Create a model of one of the solids
made of known cross-sections.

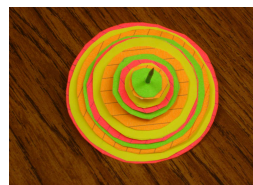
Nov 15-9:37 PM

Write a paragraph or two that addresses these questions and other comments that you have.

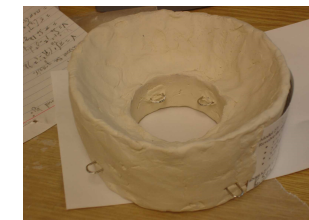
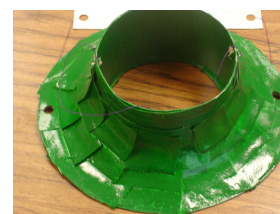
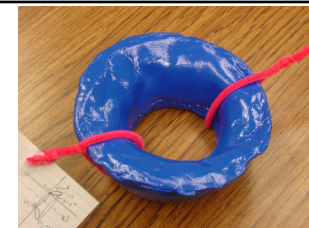
- How did this lab help you with seeing how the figures were made and how volume could be calculated?
- What observations can you make about these types of figures?
- Is there a general formula for finding the volume of a figure of this type?
- What has to be true of the formulas for the areas under the curve if they are to be used in an integral to find volume?

Nov 15-7:11 PM

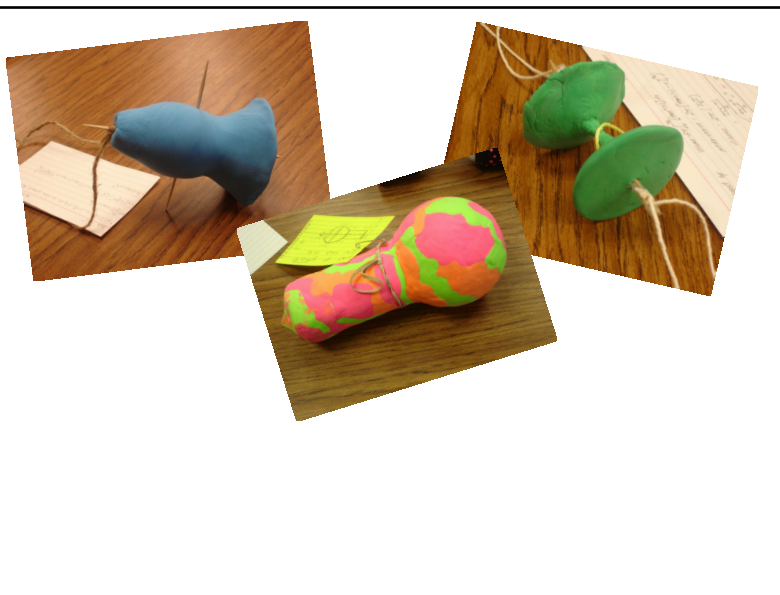
Student Made Models of Solids of Revolution



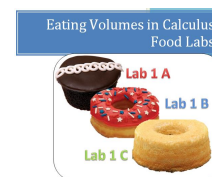
Nov 15-5:58 PM



Jan 24-12:56 AM



Jan 24-12:56 AM



Files are available at




<http://greenapples.wikispaces.com/Calculus>

Jan 24-8:23 AM

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Resources

Attachments

cupcake and donut gsp files.gsp

FoodLab#1DisksandWashers 2009.doc

FoodLab1cShells2009.doc

Fun with Integrals revised 2009 answers.doc

Fun with Integrals revised 2009.doc

Volumes on Base.gsp

Area Cross-SectionLab and Worksheet.doc

Area with known cross sections LabKey.doc